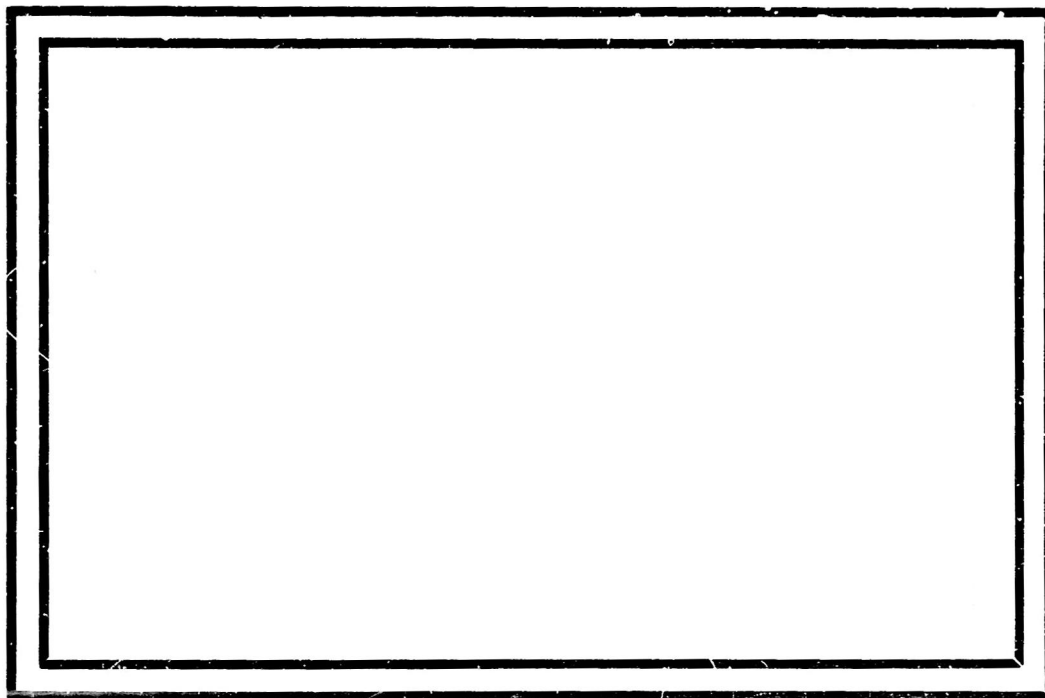# University of Maryland

PHYSICS DEPARTMENT

College Park, Maryland

NUCLEAR SHELL MODEL FT-VALUES for INTERMEDIATE COUPLING

R. Schulten and R. A. Ferrell[*]

Max-Planck-Institut fuer Physik,   Gottingen, Germany

and the

University of Maryland, College Park, Maryland

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Nuclear Shell Model ft-Values for Intermediate Coupling

R. Schulten and R. A. Ferrell*,
Max-Planck-Institut fuer Physik, Goettingen, Germany and the
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Using the wave functions already calculated by one of us for various light nuclei of mass number from 6 to 15[1] we have evaluated the beta-decay matrix elements, and thus the ft-values, for several of the beta-transitions among the light nuclei. These theoretical results agree satisfactorily with experiment, [2] and indicate that intermediate coupling is to be preferred as a nuclear model over strict LS or j-j coupling. [3]

The present work depends upon the derivation of the nuclear wave functions, which used the shell model with intermediate coupling. Specific assumptions were:

1) Two protons and two neutrons fill the 1s shell, forming an inert core. The remaining nucleons lie in the 1p shell, whose one-particle space wave functions,

$$r/r_0 \exp - 1/2 \, (r/r_0)^2 \times Y_1^{1, 0, -1} \quad \text{(where } r_0 = 1.7 \times 10^{-13} \text{ cm}$$

and $Y_1^{1, 0, -1}$ are the spherical harmonics), are coupled to the Pauli spin functions by a spin-orbit force whose strength is such as to produce a $j = 3/2$, $= 1/2$ splitting of 2.0 Mev for a 1p nucleon moving in the field set up by the four 1s core nucleons.

2) The interaction between nucleon pairs is a four-forces mixture weighted 0.35 Wigner, 0.35 Majorana, 0.15 Bartlett, and 0.15

Heisenberg force, with a well depth of 30 Mev and a common

radial dependence of exp - $(r/1.9 \times 10^{-13} cm)^2$. The spin-orbit

force of 1) also acts between all nucleon pairs. The tensor force

is neglected.

3) The wave functions contain no admixtures from higher shells --

the energy is diagonalized wholely within the 1p, (that is,

$P_{3/2}$ and $P_{1/2}$), shell.

The above assumptions led to wave functions yielding magnetic

moments in agreement with experiment to the accuracy with which

3) can be expected to hold, (~ 5 per cent). Using the same wave functions

to calculate the matrix elements in log ft = C - log ($\int 1 \ ^2 + \int \sigma \ ^2$),

(where we take the Fermi and Gamow-Teller coupling constants equal,

and fit the constant C to the neutron ft-value), we find values which are

presented in the table below and compared with those calculated on

the basis of strict j-j coupling.

β-Transition	(2) Experimental [4]	Log ft j-j coupling	intermed. coupling
a) $n \to p + e^- + \nu$	3.13	3.13	3.13
$He^6 \to Li^6 + e^- + \nu$	2.94 ± 0.04	----	2.93
$Be^7 \to Li^7 + \nu$	3.37 ± 0.01	3.43	3.41
$Be^7 \to Li^{7*} + \nu$	3.53	4.06	~~3.67~~ 3.58
a) $C^{10} \to B^{10*} + e^+ + \nu$	3.77 ± 0.20	3.43	3.43
$N^{13} \to C^{13} + e^+ + \nu$	3.67	3.59	3.69
b) $C^{14} \to N^{14} + e^- + \nu$	8.95	3.99	5.70
a) $O^{14} \to N^{14*} + e^+ + \nu$	3.52 ± 0.10	3.43	3.43
a) $O^{15} \to N^{15} + e^+ + \nu$	3.59 ± 0.03	3.61	3.61

1) R. Schulten, Doctoral Dissertation, University of Goettingen, (to be published). Zs. f. Naturforschung, 12, 759 (1953).

2) To roughly the experimental error, except for C^{14}, for which see footnote b) below.

3) For work already carried out on intermediate coupling see D. R. Inglis, Rev. Mod. Phys. 25, 390 (1953); N. Zeldes, Phys. Rev. 90, 416 (1953); A. M. Lane, Phys. Rev. 92, 839 (1953); G. E. Tauber and Ta - You Wu, Phys. Rev. 91, 443(A) (1953) and Bulletin Am. Phys. Soc. 29,1, 1A8 (1954); W. T. Sharp, H. Gellman, and G. E. Tauber, Bulletin Am. Phys. Soc. 29,1, 1A9 (1954).

4) We are endebted for information on the experimental data to R. W. King, National Research Council.

a) These entries do not represent new work. The Fermi transitions are independent of nuclear structure, while the O^{15} beta-decay is completely described by j-j coupling.

b) L-forbiddenness gives rise to a factor of ten in this matrix element, and a further partial cancellation of contributions occurs, as conjectured by Sachs, (R. Sachs, Nuclear Theory, p. 347, Addison-Wesley Pub. Co., 1953). That this is at variance with Inglis' finding, (loc. cit., p. 442), may be due to the fact that our spin-orbit force acts between all nucleon pairs, being explicitly

$$V_{s-o}(1,2) = -2.8\,Mev\left[\vec{s}_1\cdot(\vec{r}_{12}\times\vec{p}_1) + \vec{s}_2\cdot(\vec{r}_{21}\times\vec{p}_2)\right]\frac{1}{\hbar^2}e^{-r_{12}^2/(1.8\times10^{-13}cm)^2}$$

"Dressed" Particles in Quantum Field Theory.*
James L. Anderson, University of Maryland. - In time dependent per-
turbation theory there exists a certain ambiguity in the breaking up
of the Hamiltonian into two parts, one of which is to be considered
as the perturbation. If one breaks the Hamiltonian into the two parts
H_o and H_I and calls H_I the perturbing part, then it would appear equally
justified to break it into two other parts H_o' and H_I', where $H_o' = (H_o + K)$,
$(H_I' = H_o - K)$, and to call H_I' the perturbation. Since now the eigenstates
of H_o' are employed in the specification of initial and final conditions
in the theory, one might determine H_o' (or K) by requiring that it describe
properly the "dressed" particles of the theory. This means that H_o' must be
so chosen that

$$\langle \Psi_o, T[\psi(x)\psi(y)]\Psi_o \rangle = \langle \Phi_o, T[\bar{\psi}(x)\bar{\psi}(y)]\Phi_o \rangle,$$

where Ψ_o is the vacuum state of the interacting system, Φ_o the lowest
energy state of H_o', $\psi(x)$ and $\psi(y)$ are Heisenberg operators, and $\bar{\psi}(x)$ and $\bar{\psi}(y)$
are operators in an interaction representation defined by H_o'. The exis-
tence of an H_o' satisfying this condition and its relation to the renormal-
ization program will be discussed.

The Boltzmann Equation from the Statistical Mechanical Point of View.*
Melville S. Green, University of Maryland. - Recently Kirkwood and Green
have given derivations of the Boltzmann equation from the so-called
hierarchy of equations governing the distribution functions of n-uples
of molecules. The present work will exhibit a derivation which is the
first step of an expansion in powers of the concentration. If we call
φ_n the distribution of the phases of an n-uple normalized so that the
spacial distribution approaches 1, when all the molecules are far apart,
it can be shown that for a spacially uniform situation the hierarchy can
be written

$$\frac{d}{dt}\varphi_n = -c\sum_{i=1}^{n}\nabla_{p_i}\cdot\int F_{in+1}\varphi_{n+1}dp_{n+1}dx_{n+1},$$

where the right-hand side is the total time derivative along a trajectory
of an isolated system of n molecules, c the concentration, F_{in+1} the force
between the ith and the n+1th molecule, and p_i, x_i the momentum and posi-
tion of the ith molecule. These equations are solved by considering the
left-hand side to be a small perturbation. The first step in the pro-
cedure leads to the Boltzmann equation for φ_1 as well as to expressions
for φ_n which are functionals of φ_1.